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Improvements in Deep-Space Tracking by Use of Third-Order Loops

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Third-order phase-locked receivers have not yet found wide application in deep-space communications systems because the second-order systems now used have performed adequately on all past spacecraft missions. However, a survey of the doppler profiles for future missions shows that an unaided second-order loop may be unable to perform within reasonable error bounds. This article discusses the characteristics of a simple third-order extension to present second-order systems that will not only extend their doppler-tracking capability, but will widen the pull-in range, decrease pull-in time, lower the voltage-controlled oscillator noise detuning when no signal is present, and lessen the susceptibility to voltage-controlled oscillator drift.

Introduction

Second-order phase-locked receivers, used both in spacecraft and in ground tracking stations, have performed their function so satisfactorily that, up until now, there has been little or no reason to consider the installation of a more complicated system. Their performance characteristics have become well-understood, analyzable, and easily optimized relative to almost any criterion in a straightforward, well-defined way. Their capability to track incoming signals over a great range of signal levels and doppler profiles and to maintain lock and coherence at very low signal-to-noise ratios has become an accepted engineering fact.

As the more difficult deep space missions come into being, however, there is a corresponding stringency of requirements placed on the tracking instrument, as well as corresponding need to reevaluate the best ways of performing the tracking function technically, economically, and operationally. The *Mariner* Mars 1971 orbital missions and the *Pioneer F* Jupiter flyby mission, for example, have doppler rate profiles that may cause up to 30-deg steady-state phase error in the unaided second-order

loops now implemented. Such stress in receivers decreases the efficiency with which command or telemetry data are detected (by 1.25 dB at 30 deg), makes acquisition of lock difficult and faulty, and increases the likelihood of cycle slipping and loss of lock.

The way to avoid these problems is clear: eliminate or diminish the offending loop stress. This can be done by widening the loop bandwidth, by programming the uplink and downlink frequencies to correct these effects, or by increasing the order of the tracking loop. Widening the loop bandwidth increases loop noises; hence, it cannot be accepted as a general solution for the diminution of loop stress. Programming the uplink frequency and ground station local oscillator in accordance with the predicted doppler profile is certainly a valid solution, but it is costly to implement and introduces difficulty in reducing the two-way doppler data for navigation purposes. It also may require accurate predicts during critical phases of a mission where an *a priori* doppler profile is uncertain.

A third-order loop, however, will track the actual phase deviations presented to it without the need for accurate predicts. Raising the order of the loop to three would thus seem to be an ideal, even if only partial, solution because of its simplicity and possible economic factor.

The basic characteristics of third-order phase-tracking systems have been known since the first works of Jaffee and Rechtin (Reference 1), but these systems have not found wide application in the past due to what seemed to be poor acquisition and stability characteristics. Design seemed more complicated and was not well-understood. However, these potential problems have been overcome to the extent that a loop of the third order can now out-perform a second-order loop not only in its ability to track a frequency ramp with practically zero phase error, but also in its ability to acquire lock more quickly and from greater offsets. Even when synthesized with imperfect integrators within the loop filter, the third-order system will out-perform a perfect second-order system by orders-of-magnitude improvement in steady-state phase error, lock-in time, and pull-in range. One further advantage to the third-order system is that there is less of a requirement for the long time constants dictated by the high loop gains needed in the second-order loop to maintain small tracking errors.

Other advantages are that the loop filter configuration is a simple extension of presently mechanized loops, so required modification is minor; the role of the receiver operator subsequent to lock is essentially eliminated; several bandwidths are not needed to acquire rapidly; and frequency drifts in the loop voltage-controlled oscillator (VCO) cause essentially no degradation in performance. This last advantage may

remove the need to have the VCOs in ovens and thereby further extend the usefulness of the system.

Third-Order Loop Synthesis

When minimizing the total transient distortion plus noise variance by the Wiener filtering technique (Reference 1), one is led to the following optimum loop filter for tracking an input phase acceleration $\theta(t) = \Lambda_0 t^2/2$:

$$F(s) = \frac{1 + \tau_2 s}{\tau_1 s} + \frac{1}{2\tau_1 \tau_2 s^2}$$

Values are chosen so the parameter $r = AK\tau_2^2/\tau_1$ is equal to 2. In usual notation, A refers to the rms input signal amplitude and K is the loop gain. The first part of this filter resembles that used in the second-order loop (Reference 2). Based on this resemblance, one may conceive (References 3 and 4) a two-stage loop design: acquisition by the second-order loop to avoid any of the problems that a third-order system out of lock might supposedly have, and subsequent addition of the other pole to remove loop stress. As will be shown, such a configuration is particularly useful for unattended receivers; henceforth, it will be referred to as a *hybrid* design.

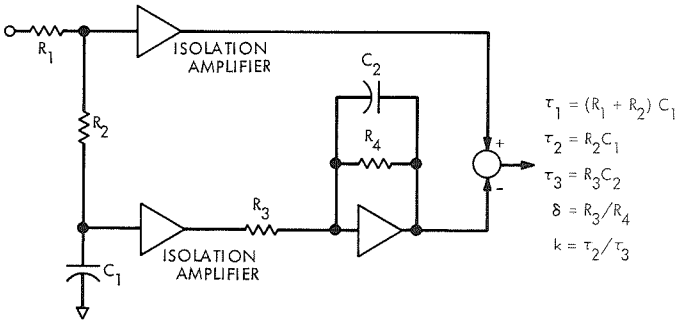
However, perfect integrators are not usually practical, so modifications are necessary. The loop filter to be considered in the remainder of this article may be synthesized in many ways, three designs of which are shown in Figure 1. These all have the same transfer function,

$$F(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s} + \frac{1}{(1 + \tau_1 s)(\delta + \tau_3 s)}$$

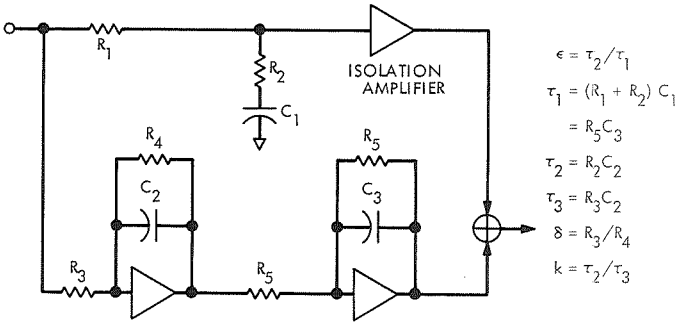
With the same loop transfer function, the designs operate identically once the loop is locked; they differ in their lock-in transient behaviors, however, because of the possibly different initial capacitor voltages. If all C's are shorted at $t = 0$, they are again identical (within limits introduced by hardware imperfections). However, when the loop is operating as a hybrid, i.e., as second order (C_2, or C_2 and C_3 shorted) during the acquisition phase and third order (C_2, or C_2 and C_3 released) after lock, Circuit I exhibits a different transient phenomenon than do Circuits II and III because of the placement of the capacitors.

In a Circuit I hybrid, the charge on C_1 at the end of the acquisition phase is related to the loop frequency mistuning, $\Omega_0 = 2\pi\Delta f$. It causes the transient shown in Figure 2 in the third-order loop, which, if too large, can knock the loop irrecoverably out of lock. Such a characteristic leads to the conclusion that *Circuit I is not generally suitable for hybrid loop design.* Circuits II and III, however, exhibit no such transient away from lock and thus make excellent hybrids.

CIRCUIT I: EXTENSION OF SECOND-ORDER LOOP BY ADDITION OF ONE INTEGRATOR



CIRCUIT II: EXTENSION OF SECOND-ORDER LOOP BY ADDITION OF TWO INTEGRATORS



CIRCUIT III: NOT AN EXTENSION OF USUAL SECOND-ORDER LOOP

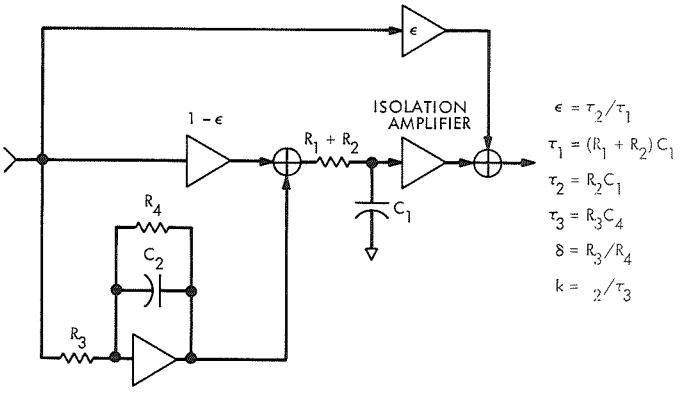


Figure 1. Third-order loop filters

The two-sided noise bandwidth in the simplified, but usual, case $\epsilon = \tau_2/\tau_1 \ll 1$ and $\delta \ll 1$ is given by

$$w_L = \frac{r}{2\tau_2}\left(\frac{r-k+1}{r-k}\right)$$

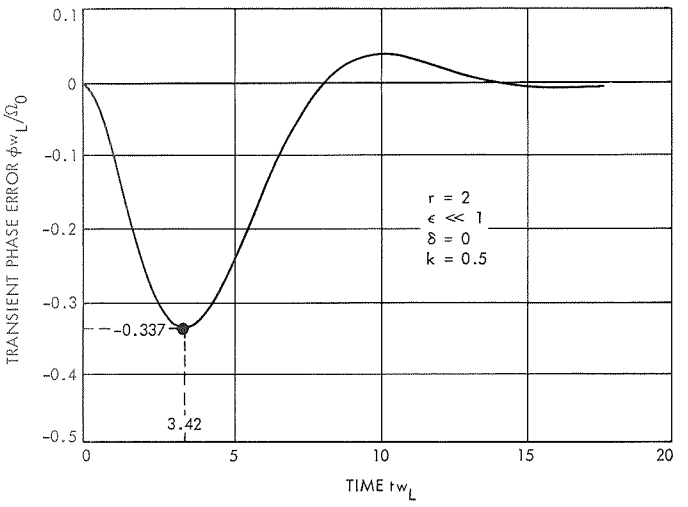


Figure 2. Circuit I hybrid loop transient caused by final lockup voltage on C_1

The parameter k is the time-constant ratio τ_2/τ_3. As compared with the loop bandwidth formula of the second-order loop, the simplified expression above is only slightly increased in complexity, and, of course, the two merge to the same result as $k \rightarrow 0$.

The primary determining factors of w_L are still τ_2 and r, just as in the second-order loop. The phase error variances of any two loops due to input noise are, of course, the same as long as their loop bandwidths are the same.

Acquisition and Lock-In Behavior

The phase-plane technique, which is useful in visualizing the lock-in behavior of second-order loops (Reference 5), does not readily extend the same advantage to third-order systems, partly because there are *three* initial conditions—phase, frequency, and frequency rate—needed to specify a unique trajectory, and partly because this trajectory lies in a three-dimensional, difficult-to-imagine, hyperplane. By analogy, however, one still can visualize that, if there is a beat frequency between the incoming sinusoid and the VCO, there will be a small dc voltage at the detector output tending to force the loop toward lock. The extra integration in the loop accumulates this force, accelerating the loop toward lock. There is thus an understandable reduction in the time required to reach the zero-beat lock-in region, and there is a corresponding increase in the loop pull-in-frequency range, as compared to that with a second-order system.

If loop damping is not set properly, the great velocity acquired by the loop phase and the momentum associated with the two integrations of the error may carry the loop frequency error past the lock region, perhaps out of lock so rapidly that recovery is not possible. Proper setting of the system roots can reduce this velocity through the zero-beat region enough to prevent frequency overshoot or irrecoverable loss of lock. In fact, if the loop has no underdamped roots, there is no transit past the zero-beat region.

The theory developed for computing the pull-in range of a second-order loop is easily extended to account for effects in the third-order loop; *there is an enhancement in the acquisition range by approximately the square root of the added integrator dc gain.* In fact, experimental evidence (Figure 3) verifies this formula exceedingly well all the way out to the point where IF filtering or minute equipment bias imperfections begin to limit the loop pull-in.

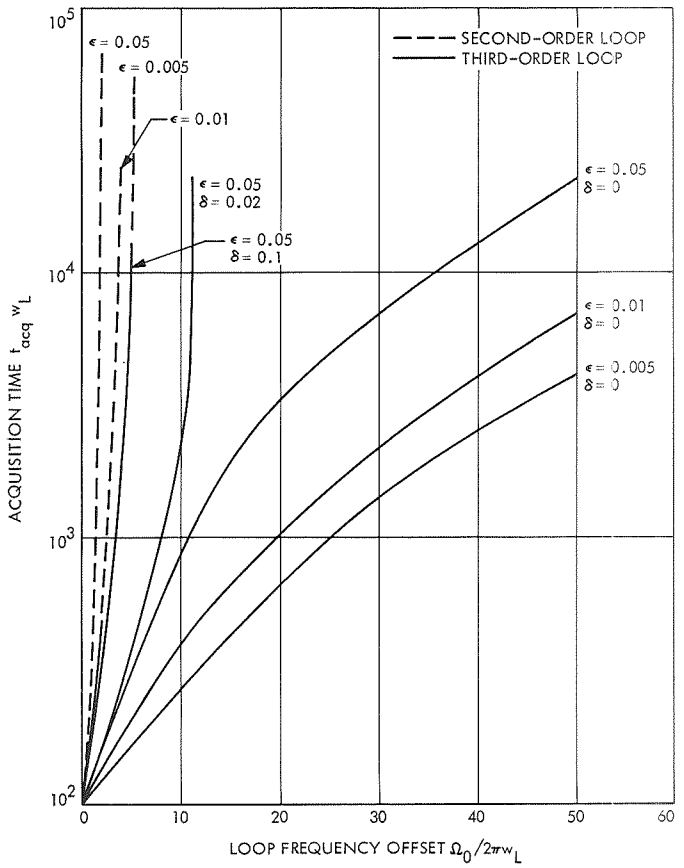


Figure 3. Measured acquisition time as a function of loop frequency offset (normalized axes)

Steady-State Errors

Due to imperfect integrations, there are steady-state phase errors that build up because of the instantaneous frequency detuning and the rate of change of this detuning. Compared with the corresponding expression for a second-order loop, the steady-state error ϕ_{ss} due to an instantaneous frequency offset is reduced by a factor of about δ, and the error caused by a frequency rate is diminished by a factor of about $\delta + (\epsilon/k)$. Such a comparison reflects the desirability not only of making ϵ and δ very small, but also of keeping k as large as other factors will permit. Figure 4 depicts the steady-state phase errors that result from frequency offsets and frequency rates.

Root Loci

For a given set of loop constants, the loop gain K or signal level A may be varied and the position of the poles of the transfer function $L(s)$ plotted. Since $r = AK\tau_2^2/\tau_1$ is proportional to both A and K, it may be used as the independent variable. The system roots start at the poles of $F(s)$ at $r = 0$ and finally terminate at the zeros of $F(s)$. These loci take on different characteristics, depending on the value of $k = \tau_2/\tau_3$.

The six loci illustrated in Figure 5 show, for various increasing k:

(a) When $k > k_{max}$, there are two underdamped (complex) and one overdamped (real) roots for all $r > r_{osc}$.

(b) When $k = k_{max}$, there are two underdamped and one overdamped roots for all $r > r_{osc}$ except at $r \approx 3$, at which point all three roots become equal.

(c) When $k_0 < k < k_{max}$, there is a region where two roots pass from underdamped to critically damped, to overdamped, to critically damped, and finally to underdamped.

(d) At $k = k_0$, the system roots are always critical or overdamped for $r > 3.375$.

(e) The $k < k_0$ case is similar to the $k = k_0$ case, except there is a root nearer the origin, indicating a more sluggish response when $k < k_0$.

(f) When $k = 0$, the zero cancels the pole near the origin, producing a second-order loop.

The cases illustrated in Figure 5b and 5d are of special interest. Figure 5b depicts the maximum value of k (i.e., k_{max}) that can be used when no underdamped roots are desired. In such a design, there is only one fixed operating signal level (i.e., $r = 3$). Figure 5d shows the maximum

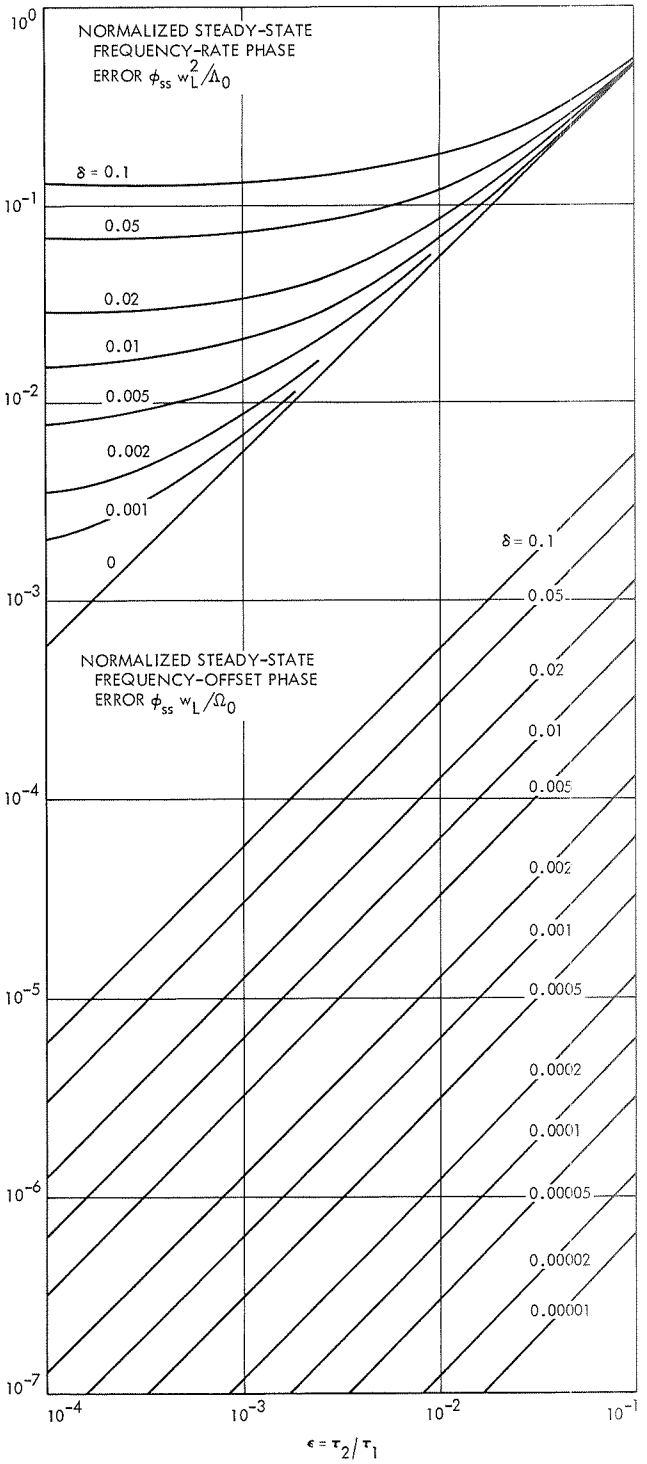


Figure 4. Normalized phase errors due to frequency offsets and frequency rates as functions of loop filter imperfections

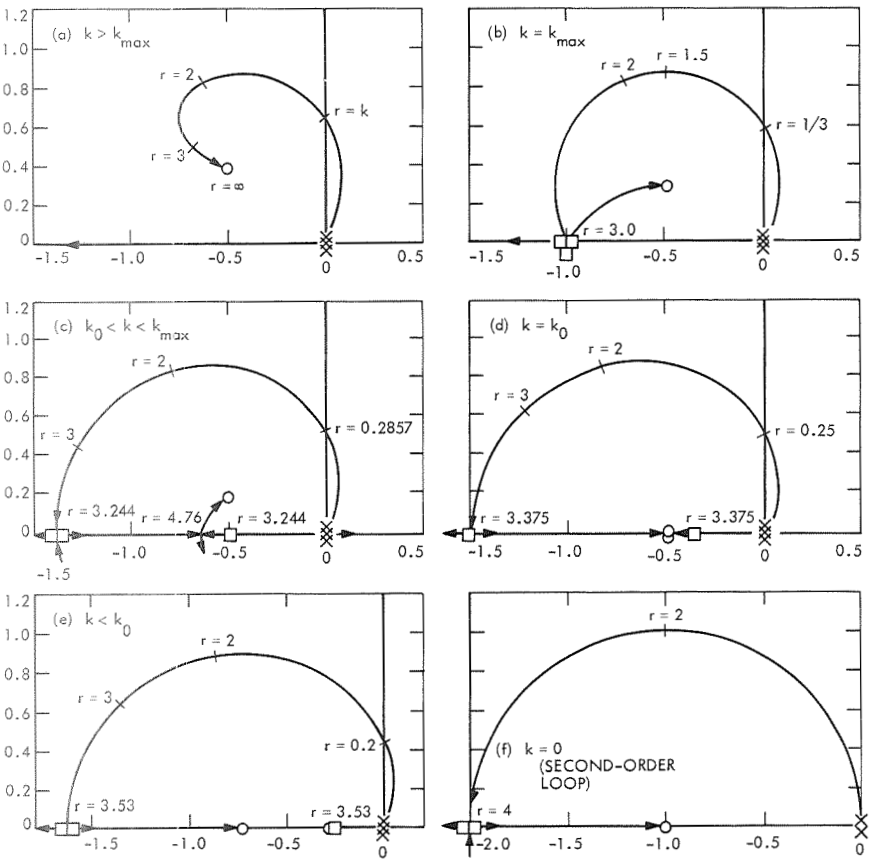


Figure 5. Root loci of the third-order loop as a function of $r = AK\tau_2^3/\tau_1$ for various values of the parameter $k = \tau_2/\tau_3$

value of k (i.e., k_0) that can be used if no underdamped roots are desired at any signal level above a design point producing $r_0 = 3.375$.

To minimize the possibility that acquisition is faulty, it is necessary that damping be critical or beyond; to minimize steady-state error once lock is achieved, k should be as large as possible. These two conditions are met in slightly different ways according to the type of signals to be tracked. If the design is to be for signals of a fixed level, then k should be set to k_{max} and r should be set to produce critical damping at this level (Figure 5b). If the design is to be for signals of various intensities, then k should be made equal to k_0 and r should be set for critical damping (Figure 5d) at the weakest expected signal level to ensure that the roots are never underdamped.

The gain margin is the ratio r/r_{osc}, which is about 22 dB for variable signal level designs, 19 dB for fixed signal level designs and 12 dB for hybrid designs.

Effect of Internal and VCO Noise

The effect of VCO and other noises internal to the loop can be modeled as an equivalent noise voltage, $n_v(t)$, appearing at the VCO input. The form of the phase-error it causes greatly resembles the corresponding equation for second-order loops, but the phase error variance is about 10 to 18% higher than that for the second-order case. Hence, there is no relaxation in the requirement for spectral purity in the VCO to be used. However, there are other effects in the VCO not well modeled spectrally; one such effect is a steady drift in rest frequency due to some change in the oscillator operating condition, such as temperature or bias voltage. These appear to the loop error detector as slight alterations to the frequency offset or rate of the incoming sinusoid. Such effects can be analyzed as part of the loop overall transient. Because the third-order loop minimizes the effect of such transients, the drift requirement on VCOs may be greatly relaxed.

Noise-Detuning of the VCO (Out of Lock)

When acquisition begins with the loop filter capacitors having the initial random charges as deposited in them by input noise prior to application of signal, there are random voltages which deviate the VCO from its rest frequency by perhaps $1/\delta$ times as far as that in a second-order loop with the same loop gain K. This comparison is somewhat unfair, as it fails to recognize the increased tracking capability of the third-order system. To judge performance between second- and third-order systems fairly, it is necessary to raise the gain of the second-order loop by this same $1/\delta$ to equate the static phase errors due to detuning. (There will be little change in the second-order loop's ability to track

an acceleration, however.) The frequency wanderings of the second-order system will then *always exceed those of the third-order system* by at least a factor $1 + (\epsilon/k\delta)$. When $k\delta$ is smaller than $\epsilon = \tau_2/\tau_1$, the advantage is obvious.

Concluding Remarks

This summary article has indicated the tracking performance gains that can be obtained by simple modifications or redesign of existing receivers. The interested reader desiring more detail may consult Reference 6, which develops the theoretical performance of practical third-order tracking systems; presents design methods and procedures by which they may be synthesized; discusses hardware configuration and implementation factors; and indicates, by actual observed data, that the design goals and performance measures are as specified by the theory.

References

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